SKILLS & APPLICATIONS TASK – TRIAL TEST A

Exponentials & Logarithms

STUDENT DETAILS:

NAME: ....*Answers*...........................

HOME GROUP: ....

Assessment Conditions:

written (supervised)   60 min   Only formula sheet given (NO NOTES)

Assessment Criteria:

Concepts and Techniques

CT1   Knowledge and understanding of concepts and relationships.

CT2   Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.

CT3   Application of mathematical models.

CT4   Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

RC1   Interpretation of mathematical results.

RC2   Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.

RC3   Use of appropriate mathematical notation, representations, and terminology.

RC4   Communication of mathematical ideas and reasoning to develop logical arguments.

RC5   Development and testing of valid conjectures.

Assessment:

	Mark	Grade
OVERALL		

Exponentials & Logarithms

QUESTION 1   (X marks)

a) Simplify:

    i.) $y^3 y^5$ y^8

    ii.) $\left(3x^3\right)^4$ $81x^{12}$

    iii.) $x+x+x+x+x$ $5x$

    iv.) $b+b+b+b\times b\times b\times b$ $3b+b^4$

    v.) $\left(\dfrac{5a^2}{2}\right)^3 \;=\; \dfrac{125a^6}{8}$

    vi.) $3h^3 \times 4h^6 \;=\; 12h^9$

    vii.) $\dfrac{16k^5}{2k^2} \;=\; 8k^3$

    viii.) $\dfrac{2p+4p^3}{2p} \;=\; 1+2p^2$

b) Write as a single power of 2:

    i.) $\dfrac{1}{4} \;=\; \dfrac{1}{2^2} \;=\; 2^{-2}$

    ii.) $\sqrt{8} \;=\; (8)^{1/2} \;=\; \left(2^3\right)^{1/2} \;=\; 2^{3/2}$

    iii.) $\dfrac{1}{2\sqrt{2}} \;=\; \dfrac{1}{2 \cdot 2^{1/2}} \;=\; \dfrac{1}{2^{3/2}} \;=\; 2^{-3/2}$

    iv.) $\sqrt[5]{16} \;=\; (16)^{1/5} \;=\; \left(2^4\right)^{1/5} \;=\; 2^{4/5}$

Exponentials & Logarithms

QUESTION 2　　　(X marks)

a)　　Clearly show the following graph can be used to approximate the value of each of the following logarithms:
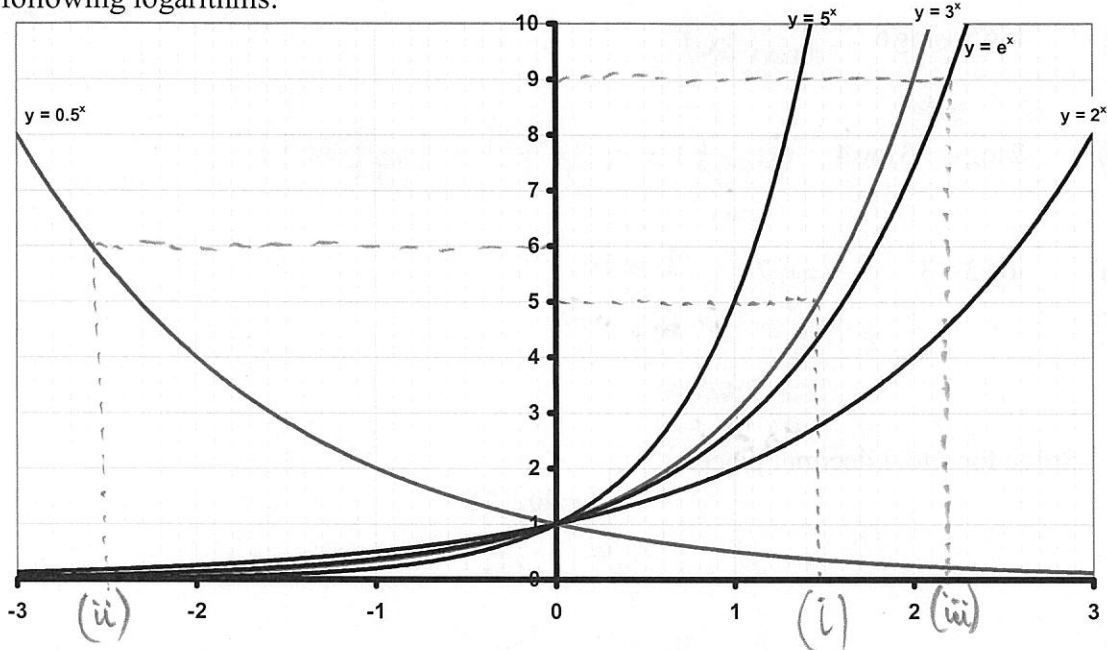
i)　　　$\log_3 5 \approx 1\cdot 4$

ii)　　$\log_{0.5} 6 \approx -2\cdot 5$

iii)　　$\ln 9 \approx 2\cdot 2$

b)　　Find the value of x in each of the following cases:

i)　　　$x = \log_2 8$　　$2^x = 8 \Rightarrow x = 3$

ii)　　$\log_x 36 = 2$　　$x^2 = 36 \Rightarrow x = 6$

iii)　　$\log_5 x = 3$　　$5^3 = x \Rightarrow x = 125$

iv)　　$x = \log_3 3^{0.3}$　　$3^x = 3^{0\cdot 3} \Rightarrow x = 0\cdot 3$

v)　　$\log x = 4$　　$10^4 = x \Rightarrow x = 10,000$

vi)　　$\ln x = 2$　　$e^2 = x$

vii)　　$\log 1 = x$　　$10^x = 1 \Rightarrow x = 0$

viii)　　$\ln e^x = 4$　　$e^4 = e^x \Rightarrow x = 4$

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QUESTION 3   (X marks)

a)   Write as a single logarithm:

    i) $\log 5 + \log 6$ $\log(30)$

    ii) $2\log 4 + 3\log 4$ $5\log 4 = \log(4^5) = \log(1024)$

    iii) $\log 2 + 3$ $= \log 2 + \log 10^3$
$$= \log 2 + \log 1000$$
$$= \log 2000$$

b)   Solve for x to 4 decimal places:

$$x = \log_3 15$$

$$3^x = 15$$
$$\log 3^x = \log 15$$
$$x \log 3 = \log 15$$
$$x = \frac{\log 15}{\log 3} = 2.4650$$

c)   Solve for x to 4 decimal places:

$$235 = 46(1.15)^x$$

$$\frac{235}{46} = 1.15^x$$
$$\log\left(\frac{235}{46}\right) = \log(1.15^x)$$
$$\log\left(\frac{235}{46}\right) = x \log 1.15$$
$$x = \frac{\log\left(\frac{235}{46}\right)}{\log 1.15} = 11.6694$$

d)   Solve for x to 4 decimal places:
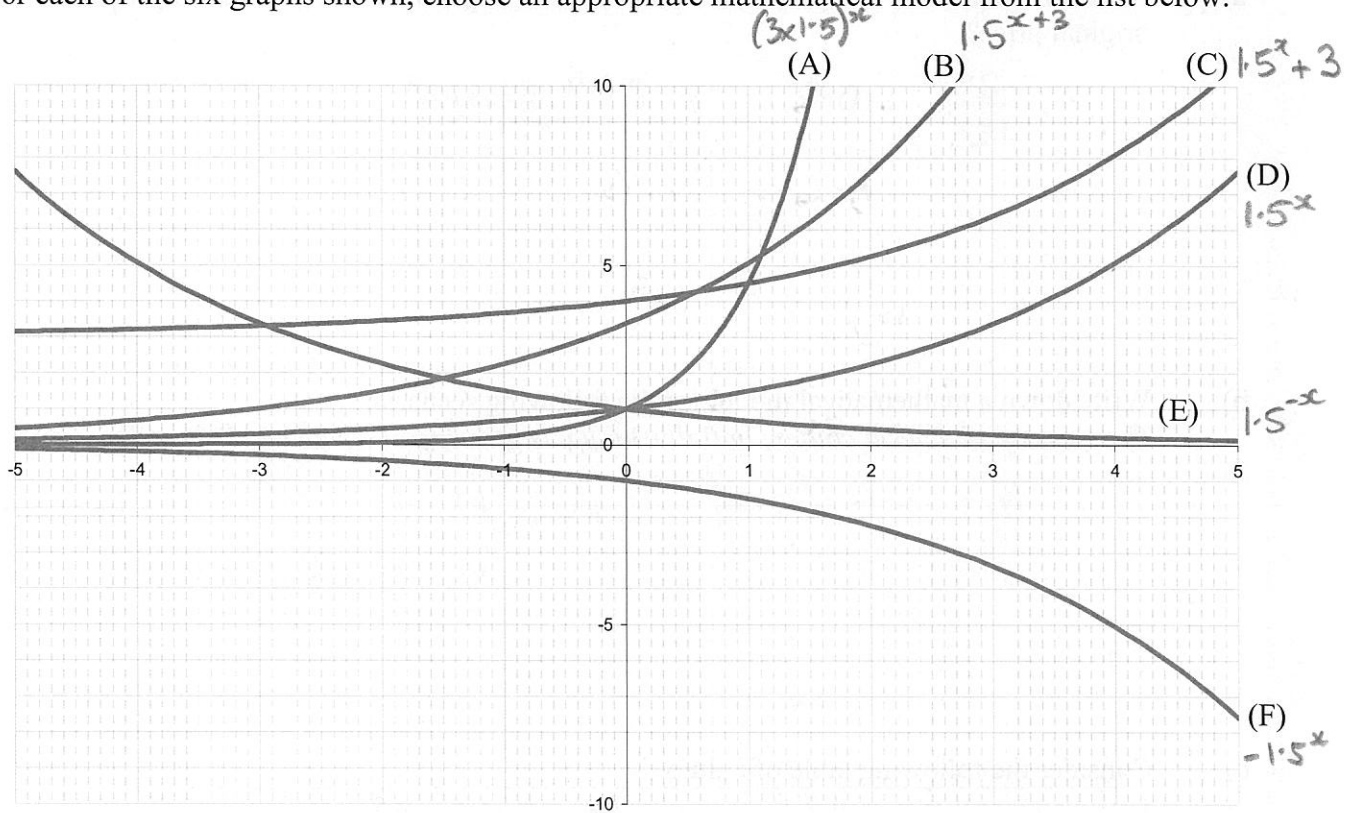
$$1.56 = 6.4x^{0.18}$$

$$\frac{1.56}{6.4} = x^{0.18}$$
$$\left(\frac{1.56}{6.4}\right)^{1/0.18} = x$$
$$x = 0.0001$$

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QUESTION 4   (X marks)

For each of the six graphs shown, choose an appropriate mathematical model from the list below.

$(3 \times 1.5)^x$ (A) 1.5^{x+3} (B)   (C) $1.5^x + 3$

(D) 1.5^x

(E) 1.5^{-x}

(F) -1.5^x

Choose from this list of equations:

$$y = 1.5^x \quad \text{(D)}$$
$$y = -1.5^x \quad \text{(F)}$$
$$y = 1.5^{-x} \quad \text{(E)}$$
$$y = (3 \times 1.5)^x \quad \text{(A)}$$
$$y = 1.5^{x+3} \quad \text{(B)}$$
$$y = 1.5^{x-3}$$
$$y = 1.5^x + 3 \quad \text{(C)}$$
$$y = 1.5^x - 3$$

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QUESTION 5   (X marks)

a)   If an item is marked-up by 25%, what discount is required to match the final price with the original price?

$$\frac{25}{125} = 0.2 \quad \text{ie} \quad \underline{20\% \text{ discount.}}$$

b)   Write down a mathematical model for the following scenario.
   "The mass of a bacteria colony, initially 3.6 µg, is growing at a rate of 12% per hour."

$$M = 3.6 \times (1.12)^h \quad, \text{where } M = \text{mass of colony (µg)}$$
$$h = \text{number of hours.}$$

c)   Consider the following table of values.

t	1.2	2.8	4.5	7.2
A	17.36	3.19	1.23	0.48

   Select the most appropriate model from the list below.
   a. $A = 0.958t^2 - 10.6t + 27.9$
   b. $A = 22e^{-0.576t}$
   c. $A = 25t^{-2}$ Power
   d. $A = 16.8 - 9.6\log t$

   Justify your selection.

   $r^2 \approx 1$, which is better than the other models.

d)   Mr Borrow purchased a car for $25,000 and wants to keep track of his car loan. If he borrowed the full amount at a rate of 10.4% p.a. and his bank charges interest monthly, complete the following spreadsheet by entering appropriate *formulae*:

	A	B	C	D	E
1	Month	Start of Month ($)	Interest in month ($)	Payment ($)	End of month ($)
2	1	$25,000	=0.104/12 * B2	(enter here)	= B2 + C2 − D2
3	2	= E2	=0.104/12 * B3	(enter here)	= B3 + C3 − D3

6

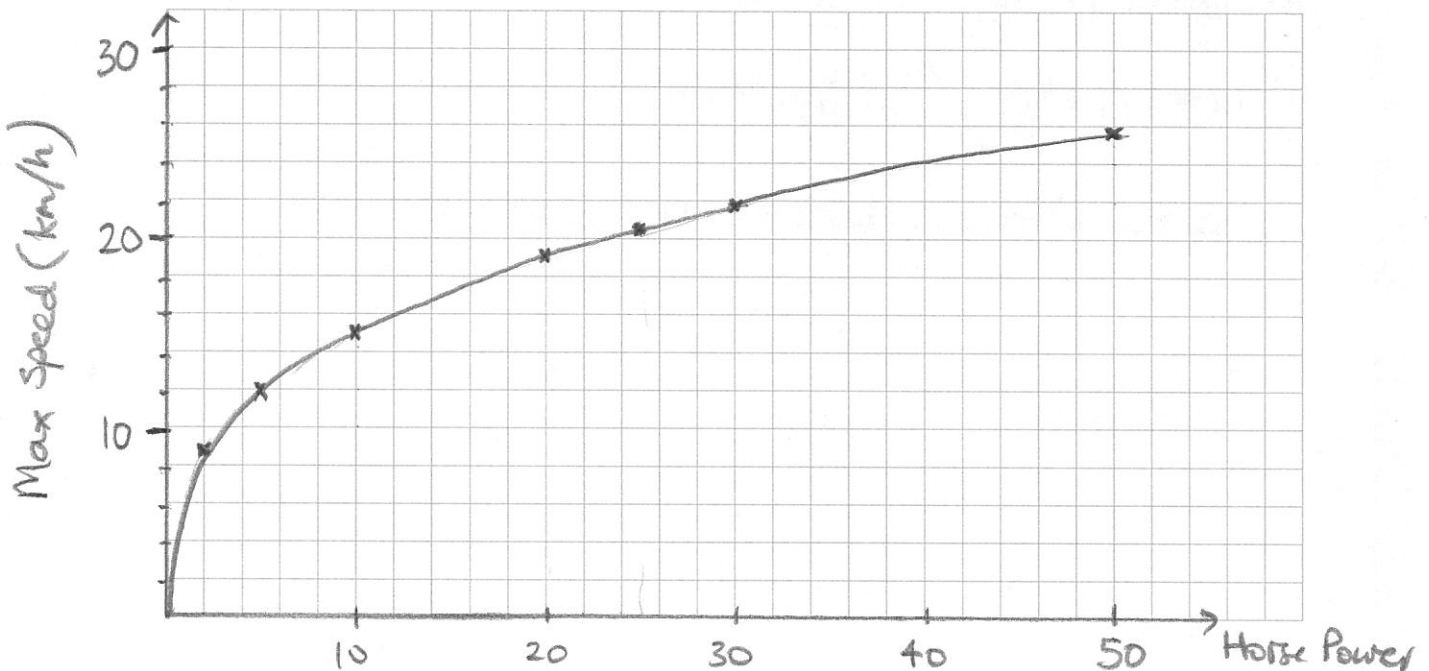# Exponentials & Logarithms

QUESTION 6   (X marks)

The following data was obtained for a series of engines (for a small boat).

Horse Power	2	5	10	20	25	30	50
Max Speed (km/h)	8.8	12.0	15.1	19.0	20.5	21.8	25.8

Sketch an appropriate graph of this data.

The following model was considered most appropriate:

$S = 6.99(hp)^{0.334}$, where S is the maximum speed in km/ and hp is the number of horse power.

a) Use the model to predict the maximum speed of a 15 horse power engine.

$\left[\text{G-solv, Y-calc} \right]$ 17.3 km/h

b) Comment on the expected accuracy of your prediction.

Fairly accurate as we are interpolating.

c) Use the model to predict the maximum speed of a 100 horse power engine.

$\left[\text{G-solv, Y-CALC (using different scale to capture 100hp)} \right]$ 32.5 km/h

d) Comment on the expected accuracy of your prediction.

Low accuracy as we are extrapolating a long way.

e) Use the model to suggest the minimum engine size required for a top speed of 27 km/h.

$\left[\text{G-solv, X-calc} \right]$ 57.1 ⇒   58 hp.

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QUESTION 7   (X marks)

a)   Compare the interest earned on an investment of $15,000 for 2 years at 6.8% p.a. if:

   A) The interest is compounded yearly
   B) The interest is compounded daily

A) $15000 \times (1.068)^2 = 17,109.36$

B) $15000 \times \left(1 + \frac{0.068}{365}\right)^{2 \times 365} = 17,185.01$

$\therefore$ Option B) earns $75.65 more interest than A).

b)   Mr Brown sold a tractor for $40,000 after 6 years ownership. If the rate of depreciation was 14% p.a., approximate the amount he originally paid for the tractor.

$40000 = A_0 \times (0.86)^6$

$A_0 = \dfrac{40\,000}{0.86^6}$

$= 98,871$

ie $98,871

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c) Miss White wants her investment to increase by 50% in 3 years. What annual rate is
    required if the interest compounds:
    a. Annually
    b. Monthly

a) $x^3 = 1.5 \Rightarrow x = \sqrt[3]{1.5} = 1.145$

$\therefore 14.5\% \text{ pa. required.}$

b) $x^{36} = 1.5 \Rightarrow x = (1.5)^{1/36} = 1.0113$

i.e. 1.13% ~~pa~~ per month

$\Rightarrow 13.6\%$ p.a

d) A radioactive substance has a half-life of 40 years. Initially there is 125 μg of the substance.
    a. How much of the radioactive substance remains after 7 years?
    b. How many years will it take to decay to less than 20 μg?

a) $125 \times (0.5)^{7/40}$

$= 110.7 \, \mu g.$

$\left[\begin{array}{l} \text{\# of cycles} = \frac{7}{40} \\ (1 \text{ cycle} = 40 \text{ years}) \end{array} \right]$

b) $20 = 125 \times 0.5^n$

$\frac{20}{125} = 0.5^n$

$\log\left(\frac{20}{125}\right) = \log(0.5)^n$

$\log\left(\frac{20}{125}\right) = n \log(0.5)$

$n = \dfrac{\log\left(\frac{20}{125}\right)}{\log(0.5)}$

$n = 2.64 \text{ cycles}$

$\therefore$ Number of years $= 2.64 \times 40$

$= 105.8$

$\therefore$ 106 years